Teacher notes
Topic A

Cue hitting a snooker ball.

There has been some discussion of a problem in the textbook in chapter A5: A cue hits a snooker ball horizontally at a height *h* from the center of the ball. Show that when $h = \frac{2R}{5}$, the ball begins to roll without slipping. What is the role of friction in this problem?

Suppose that, after hitting the ball with the cue, the linear speed of the ball is *u*. The impulse provided to the ball is *mu*. The impulse is created by the cue exerting a force on the ball. Initially the ball is at rest so there is no issue of a frictional force acting. If the cue is in contact with the ball for a time δt then:

$$J = mu = F\delta t$$

The force *F* provides the torque to rotate the ball:

$$\tau = Fh = \frac{\delta L}{\delta t} \Rightarrow Fh\delta t = \delta L = \frac{2}{5}mR^2\omega$$

Here, ω is the angular speed after time δt.

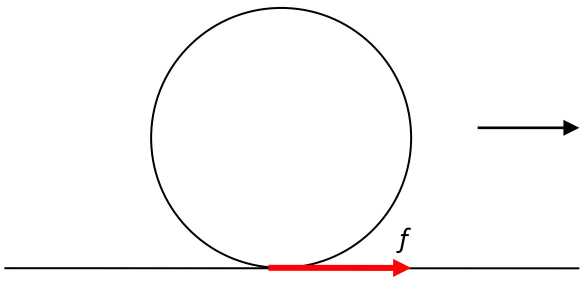
Using the first equation

$$muh = \frac{2}{5}mR^2\omega \Rightarrow \omega = \frac{5uh}{2R^2} = \frac{u}{R} \times \frac{5h}{2R}.$$

This shows that if $h > \frac{2R}{5}$, the ball rotates at an angular speed faster than the one required for rolling without slipping (i.e., $\frac{u}{R}$) and if $h < \frac{2R}{5}$ it rotates slower than required. On the other hand, if $h = \frac{2R}{5}$, then $\omega = \frac{u}{R}$ which is the condition for rolling without slipping. In this case, no friction is required; the point of contact of the ball with the ground is instantaneously at rest, there is no sliding and no reason for a frictional force to develop. (This is unlike the motion of the ball down an inclined plane where friction is necessary to provide the torque for rotating the ball. Here the ball is already rotating, and no additional torque is required.)

But if either of $h > \frac{2R}{5}$ or $h < \frac{2R}{5}$ holds then there is sliding, and a friction force will develop.

Suppose that $h > \frac{2R}{5}$. Then a frictional force acting in the direction of velocity will develop which will have the effect of reducing the angular speed of the ball. In this case:



$$\tau = -I\frac{d\omega}{dt}$$
$$fR = -\frac{2mR^2}{5}\frac{d\omega}{dt}$$
$$\mu_k mgR = -\frac{2mR^2}{5}\frac{d\omega}{dt}$$
$$\frac{d\omega}{dt} = -\frac{5\mu_k g}{2R}$$
$$\omega = -\frac{5\mu_k g}{2R}t + \frac{u}{R}\times\frac{5h}{2R}$$

The linear speed will be $v = u + \mu_k gt$ and so the angular speed will be reduced to the speed required for rolling without slipping after a time given by

$$v = \omega R$$
$$u + \mu_k gt = (-\frac{5\mu_k g}{2R}t + \frac{u}{R}\times\frac{5h}{2R})R$$
$$t = \frac{(5h-2R)u}{7\mu_k gR}$$

Notice that if $h = \frac{2R}{5}$, this time is zero; the ball immediately starts rolling without slipping after being hit by the cue as we already know.

The speed after this time is

$$v = u + \mu_k g t = u + \mu_k g \frac{(5h-2R)u}{7\mu_k g R}$$
$$= u + \frac{(5h-2R)u}{7R}$$
$$= \frac{7Ru + 5hu - 2Ru}{7R}$$
$$v = \frac{5(R+h)u}{7R}$$

The distance travelled in this time is

$$s = ut + \frac{1}{2}at^2$$
$$= u \times \frac{(5h-2R)u}{7\mu_k g R} + \frac{1}{2}\mu_k g \left(\frac{(5h-2R)u}{7\mu_k g R}\right)^2$$
$$= \frac{5u^2(5h-2R)(R+h)}{49\mu_k g R^2}$$

And so, the work done by *f* is

$$W = fs$$
$$= \mu_k m g \times \frac{5u^2(5h-2R)(R+h)}{49\mu_k g R^2}$$
$$= \frac{5mu^2(5h-2R)(R+h)}{49R^2}$$

Again, we see the factor of $(5h-2R)$ which implies no work done when $h = \frac{2R}{5}$.